A WAY TO DEAL WITH MODEL-PLANT MISMATCH FOR A RELIABLE DIAGNOSIS IN TRANSIENT OPERATION

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ABSTRACT

Least-squares health parameter identification techniques such as the Kalman filter have been massively used to solve the problem of turbine engine diagnosis. Indeed, such methods give a good estimate provided that the discrepancies between the model prediction and the measurements are zero-mean, white random variables. In turbine engine diagnosis, however, this assumption does not always hold due to the presence of biases in the model. This is especially true for transient operation. As a result, the estimated parameters tend to diverge from their actual values which strongly deteriorates the diagnosis.

The purpose of this contribution is to present a Kalman filter diagnosis tool where the model biases are treated as an additional random measurement error. The new methodology is tested on simulated transient data representative of a current turbofan layout. While relatively simple to implement, the newly developed diagnosis tool exhibits a much better accuracy than the original Kalman filter in the presence of model biases.

NOMENCLATURE

\( \hat{a} \) estimation of an unknown variable  
\( a \) nozzle exit area (nominal value : 1.4147 m\(^2\) )  
\( b_k \) the mean model bias  
BCM Bias Compensation Module  
DEKF Dual Estimation Kalman Filter  
hpc high pressure compressor  
hpt high pressure turbine

\( k \) discrete time index  
lpC low pressure compressor  
lpt low pressure turbine  
\( p_i^0 \) total pressure at station \( i \)  
\( R_{b,k} \) covariance matrix of the model bias  
SE\( i \) efficiency degradation of the component whose entry is located at section \( i \) (nominal value : 1.0)  
\( T_i^0 \) total temperature at station \( i \)  
TI Transient Index  
\( u_k \) actual command parameters  
\( v_k \) actual external disturbances  
\( w_k \) actual but unknown health parameters  
\( x_k \) actual but unknown state variables  
\( y_k \) observed measurements  
\( \varepsilon_k \) measurement noise vector  
\( \nu_k \) process noise vector  
\( \mathcal{N}(m,R) \) A Gaussian probability density function with mean \( m \) and covariance matrix \( R \)

INTRODUCTION

The diagnosis tool considered herein is basically a gas path analysis method whose purpose is to assess the deviations of some health parameters on the basis of measurements collected within the gas path of the engine [1]. The health parameters are coefficients affecting the efficiency and the flow capacity of the components (fan, lpc, hpc, hpt, lpt, nozzle) while the measurements are inter-component temperatures, pressures as well as ro-
tational speeds. The health assessment leads to a diagnosis of the engine condition which allows suitable maintenance actions to be undertaken.

The health parameter estimation is achieved by a Kalman filter which is a minimum mean square error (variance) estimator within a recursive framework [2]. This means that the estimated health parameters minimise the distance (in the least squares sense) between a model prediction and the observed measurements. Moreover, the recursive structure of the Kalman filter updates the values of the identified health parameters as new data are available which is a useful property in real-world applications such as on-board performance monitoring [3].

Since the first research efforts of Urban [4], most of the gas path analysis methods were restricted to measurements observed under steady-state operation of the engine, mainly for computational load reasons. For a few years, the ability to extract the engine condition from transient data has been investigated using various techniques such as least-squares estimation [5, 6], artificial neural networks [7] for batch treatment of the data, or Kalman filters [8–11] in a recursive framework.

More specifically, as illustrated in [10] by the authors, the use of measurements representative of transient behaviour is known to significantly improve the diagnosis procedure provided that a faithful dynamic model is available. Indeed, transient operation allows a much greater number of operating points to be considered, thereby increasing the analytical redundancy.

Although the existence of a perfectly faithful model is generally assumed, this hypothesis is rarely met in practice. In fact, complex phenomena such as heat transfers, volume dynamics, clearances or airflow and power off-takes are poorly or not modelled in current state-of-the-art aero-thermodynamic engine models [12, 13]. Consequently, the performance predictions generated by the dynamic model are biased with respect to measurements taken on the engine. As reported in [11], those biases strongly spoil the efficiency of the diagnosis tool.

The present contribution proposes a solution to the model biases by considering them as an additional measurement error. Indeed, from the point of view of an external observer, it is not possible to distinguish a model bias from a sensor error. However, unlike sensor errors which are basically unpredictable beforehand, the model biases of interest have a more predictable nature which can be studied by comparing the model outputs and the measurements observed on a healthy engine during a learning phase previous to the health parameter assessment.

**DESCRIPTION OF THE METHOD**

The scope of this section is to provide a short description of our diagnosis tool and to present the methodology we have developed to cope with model-plant mismatch and its integration within the diagnosis algorithm.

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1 Linearised models are often used to lower the computational burden.

2 In addition, $\nu_k$ and $\epsilon_k$ are assumed uncorrelated.
Dealing with Model Biases

The physical meaning of the identified health parameters is only valid provided that the model is faithful. Otherwise, as reported by Volponi in [11], the health parameters become “tuners” which are adjusted by the identification process to fit the behaviour of the real engine, loosing sense for diagnosis reports. Indeed, as formulated in the preceding section, the Kalman filter assumes that the discrepancies between the model and the measurements are zero on average. This means that it is able to cope with isolated errors of high magnitude (specified by the covariance matrix) provided that these errors are zero on average (i.e. typically on 20 to 30 successive measurement samples in the application of interest).

Unfortunately, model biases do not have a stochastic nature and vary relatively slowly (i.e. they are constant on time bases of 20 to 30 samples). This introduces errors whose mean is different from zero which perturbs the health parameters identification. In practice, it has been observed that model biases as small as the measurement noise standard deviation can dangerously reduce the reliability of the diagnosis (see the further application). Therefore the model biases have to be considered as an additional part of the engine model. In reality, the model-plant mismatch reflects the approximations made in both the state transition equation (1) and the measurement equation (2). A possible mathematical translation of this fact is to consider that neither $e_k$, nor $v_k$ are zero-mean, Gaussian random variables.

In this contribution, we propose to gather all the effects caused by modelling errors on the single measurement equation, namely:

$$y_k = G(u_k, v_k, w, x_k) + e_k$$

where the measurement noise $e_k$ is now seen as a hybrid bias-noise term gathering the measurement noise and the model-plant mismatch. That is, the bias is considered as a Gaussian random variable of variable mean. Two reasons conducted this choice: first, the Kalman filter deals with this type of random variables and second, a Gaussian random variable is totally defined by its mean and its standard deviation (which is simple to handle in practice). This is written mathematically as:

$$e_k = N(b_k, R_{b,k})$$

Provided that $b_k$ and $R_{b,k}$ are known, the mechanism of the Unscented Kalman filter can be applied by making the following substitutions:

$$r_k = y_k - \hat{y}_k \quad \rightarrow \quad r_k = y_k - \hat{y}_k - b_k$$
$$R_y \quad \rightarrow \quad R_{b,k}$$

**Determinatation of $b_k$ and $R_{b,k}$**

As already mentioned above, the model biases are not strictly speaking random variables. Indeed, they can be studied beforehand, for example during the acceptance tests that every engine undergoes before it is brought into service. The purpose of this off-line learning is to build a model which predicts $b_k$ and $R_{b,k}$ as precisely as possible. This approach is very close to the eSTORM philosophy [11], with the difference that in our study both the mean bias and its uncertainty are modelled, which translates in a less complex model structure (polynomial fit).
Three assumptions are made in order to simplify the determination of \(b_k\) and \(R_{b,k}\):

1. the engine steady-state model is perfectly faithful. To this end, model-matching techniques such as described in [15] may be applied
2. the engine undergoes relatively slow transient manoeuvres.
   This constraint can be expressed in terms of an upper limit on the fan rotor acceleration. This bound is application-dependent and was set here to a value of 200 RPM/s
3. during the learning phase, the engine is in healthy, nominal condition, hence the values of the health parameters are known and set to their nominal values

During this learning phase, model outputs are compared to the observed measurements without estimating the health parameters which are assumed to be at their nominal values. As the engine transient model is not perfect, the observed residuals \(r_k\) correspond to the model biases. The next step of the learning phase consists in characterising the mean \(b_k\) and the covariance matrix \(R_{b,k}\) of the observed residuals.

The problem of modelling the mean \(b_k\) is first investigated. We can reasonably suppose that the more intense the transient is, the bigger the model-plant mismatch is. Therefore, it is desirable to link the mean bias \(b_k\) to a scalar quantity which is representative of the intensity of the transient and which is easy to compute. To this end, the following transient index \(T_I\) is defined:

\[
T_I(k) = \frac{1}{N_x} \sum_{i=1}^{N_x} \frac{x_k(i) - \hat{x}_k(i)}{x_{ref}(i)}
\]

where \(N_x\) is the number of state variables of the on-board model, \(x_k(i)\) is the derivative of state variable \(i\) at time index \(k\) and \(x_{ref}(i)\) is the reference value of state variable \(i\) (e.g. at take-off rating). The units of \(T_I\) are \(s^{-1}\). The division by a reference state value is required given the order of magnitude of the different state variables. The transient index is zero in steady-state operation, positive when the engine is accelerating and negative otherwise.

The transient index is computed based on state derivatives given by the engine model rather than on actual measurements. Two reasons dictate this choice: first, the engine model produces noise-free signals and outputs directly the state derivatives; secondly, not all state variables are measurable on the real engine (e.g. the metal temperatures involved in the heat transfers), but are available in the engine model.

So, modelling the mean value of the bias comes to determine the mapping \(b = f(T_I)\) from the database of residuals. This problem is solved through a least squares polynomial fit. The selection of the polynomial order and a possible partitioning of the TI axis is a question of engineering judgement. Additional indications are provided in the application detailed further.

The mean value of the bias being modelled, the covariance matrix of the bias, \(R_{b,k}\), can now be computed. The procedure is given in algorithm 1. Depending on the partitioning adopted for the mean bias, one covariance matrix is computed per segment. As a first step, the gap between each data and the mean bias is computed for all data points belonging to a particular segment (lines 2 to 5, note that each vector \(e_i\) is a \(N_x \times 1\) vector). Finally, each element of the symmetric covariance matrix is obtained using the well-known definition of the covariance of random variables (line 6).³

### Algorithm 1 Covariance matrix computation

1. set \(N = 0\)
2. for all \(k\) such that \(T_I^{min} < T_I(k) \leq T_I^{MAX}\) do
   3. \(N = N + 1\)
   4. \(e_N = r_k - b(T_I(k))\)
   5. end for
   6. \(R_b = \frac{1}{N-1} \sum_{i=1}^{N} \{e_i e_i^T\}\)

The covariance matrix takes into account both the measurement noise (sensor inaccuracies) and the possible variability of the mean bias. Also, it should not be surprising that some off-diagonal terms of that matrix are non-zero. This simply indicates that the sensor biases are inter-dependent as the modelling errors introduce some relationships between the measurements: for instance the bias on the EGT is linked to that of low pressure spool speed, and so on.

This concludes the off-line modelling of the bias model which can now be integrated within the diagnosis algorithm in order to make it more robust to model-plant mismatch.

### Modification of the Diagnosis Algorithm

The block diagram of the modified diagnosis algorithm is shown in figure 2. A short description of the procedure is given in the sequel.

Similarly to the original procedure detailed in figure 1, the previous estimates of the state variables \(\hat{x}_{k-1}\) and health parameters \(\hat{\omega}_{k-1}\) are used together with the current inputs \(u_k\) and \(v_k\) by the engine performance model to generate an estimation of the measurements. Additionally, the transient index \(T_I(k)\) is computed using relation (7). From this transient index, the mean bias \(b_k\) and its covariance matrix \(R_{b,k}\) are obtained according to the bias model previously set up. The bias \(b_k\) is taken into account in the residual \(r_k\) which is fed into the original DEKF with the only difference that the measurement noise covariance matrix \(R_y\) is

³ in algorithm 1, the superscript \(T\) denotes the vector transpose operator
As real data were not available, we worked with simulated data only. To introduce modelling errors, we considered two different configurations of the OBIDICOTE model with regard to transient operation. Our hypothesis concerning the perfect fidelity of the steady-state model is hence implicitly satisfied.

The sensor configuration adopted in the test-cases is representative of a usual instrumentation available on modern turbofan engines and is detailed in table 1.

Table 1. SENSOR CONFIGURATION (Uncertainty is assumed to be three times the standard deviation)

<table>
<thead>
<tr>
<th>Label</th>
<th>Uncertainty</th>
<th>Label</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^0_{13}$</td>
<td>$\pm 2$ K</td>
<td>$p^0_{13}$</td>
<td>$\pm 300$ Pa</td>
</tr>
<tr>
<td>$T^0_3$</td>
<td>$\pm 2$ K</td>
<td>$p^0_3$</td>
<td>$\pm 5000$ Pa</td>
</tr>
<tr>
<td>$N_{lp}$</td>
<td>$\pm 6$ RPM</td>
<td>$N_{hp}$</td>
<td>$\pm 12$ RPM</td>
</tr>
<tr>
<td>$T^0_6$</td>
<td>$\pm 2$ K</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

“Real” Engine and On-Board Model

The OBIDICOTE model plays the role of the “real” engine. The shaft dynamics and the heat transfers in the hpc, burner and hpt are supposed to be perfectly modelled. The 7 state variables involved in this first model are listed in table 2.

Table 2. STATE VARIABLES FOR THE “REAL” ENGINE

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{lp}$</td>
<td>low pressure spool rotational speed</td>
</tr>
<tr>
<td>$N_{hp}$</td>
<td>high pressure spool rotational speed</td>
</tr>
<tr>
<td>$T_{m3b}$</td>
<td>high pressure compressor blade temperature</td>
</tr>
<tr>
<td>$T_{m3c}$</td>
<td>high pressure compressor casing temperature</td>
</tr>
<tr>
<td>$T_{m4b}$</td>
<td>combustion chamber casing temperature</td>
</tr>
<tr>
<td>$T_{m42b}$</td>
<td>high pressure turbine blade temperature</td>
</tr>
<tr>
<td>$T_{m42c}$</td>
<td>high pressure turbine casing temperature</td>
</tr>
</tbody>
</table>

Gaussian noise, whose magnitude is specified in table 1, is added to the clean measurements generated by the “real” engine model to make them closer to real ones. The sampling rate is set to 50 Hertz, which is a typical value to capture the transient effects described above.

A second model is embedded in the diagnosis algorithm and plays the role of the on-board model. This second model suffers...
from model-plant mismatch since the heat transfers are poorly modelled with respect to the “real” engine. As shown in table 3, only three state variables are involved in this second model.

Table 3. STATE VARIABLES FOR THE ON-BOARD MODEL

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N_{lp}</td>
<td>low pressure spool rotational speed</td>
</tr>
<tr>
<td>N_{hp}</td>
<td>high pressure spool rotational speed</td>
</tr>
<tr>
<td>T_{mt42}</td>
<td>high pressure turbine metal temperature</td>
</tr>
</tbody>
</table>

A first attempt of bias compensation had been made considering only the spool inertia in the on-board model, hence neglecting all heat transfer modelling. However this simple model led to too important model-plant mismatch. It was decided to include heat transfer on the hpt for two reasons: first, heat transfers are the slowest dynamics in a turbine engine and they strongly influence the transient response of the engine (see [13]); secondly, the heat transfer is placed on the hpt since it is the hottest part of the engine and therefore the thermal effects are expected to be more important than in other components, moreover the observability of the hpt thermal state is satisfactory (see [10]).

Modelling the Bias

The methodology for bias modelling described in a previous section is applied on the turbofan layout. The first step is to build a database of residuals for the healthy engine. To this end, the “real” engine and the on-board model have been subjected to random manoeuvres in open-loop between idle and take-off regime on a test bench for 2000 seconds.

The mismatch between the “real” engine and the on-board model appears in figure 4, where the normalised root mean square error (NRMSE) is plotted for each sensor. The NRMSE for the \( i \)th sensor is defined according to equation (8).

\[
\text{NRMSE}(i) = \sqrt{\frac{1}{n-1} \sum_{k=1}^{n} \left( y_k(i) - \hat{y}_k(i) \right)^2 / R_y(i,i)} \tag{8}
\]

As can be seen in figure 4, the NRMSE should be equal to one if the on-board model were perfect (white bars). Indeed, in that case, the only source of variation in the response of the “real” engine and the on-board model is the sensor noise. For the incomplete on-board model considered in this application (grey bars), it can be stated that the predictions on \( p_{13}^{0} \) and \( T_{0}^{0} \) are quite faithful; modelling errors, however, have a disastrous effect on the other measurements and particularly on \( T_{3}^{0} \). Note that the \( T_{3}^{0} \) prediction error could be reduced by introducing a heat transfer modelling in the hpc.

The delicate part of the job consists in defining a model for the residuals. Plotted in figure 5 is the cloud of \( T_{3}^{0} \) residuals with respect to TI (black circles).

After examining the residual cloud of each measurement, it was decided to split the TI axis into three distinct regions and to apply a linear least squares fit on the data to determine the mean bias (red lines in figure 5). The covariance matrices were then computed using algorithm 1. A quadratic least squares fit was also implemented, but no improvement was noted in the diagnosis results. On the other hand, higher order polynomials were rejected since the aim is to extract the global trend in the bias.

The resulting bias model is summarised in table 4. TI is a threshold value (set to \( 10^{-3} \) in this application) that makes operating points with small absolute values of TI considered as steady-state ones. For these operating points, the mean bias is set to zero and the covariance matrix \( R_{b,k} \) reduces to the original
measurement noise covariance matrix $\mathbf{R}_y$ given the assumption of a perfect steady-state model. The vectors $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$ and $\mathbf{c}_4$ are the coefficients of the linear least squares fit for the $\mathbf{T}_I > \mathbf{T}_I^*$ and $\mathbf{T}_I < \mathbf{T}_I^*$ regions respectively.

Table 4. SUMMARY OF THE BIAS MODEL

<table>
<thead>
<tr>
<th>Region</th>
<th>Mean Bias</th>
<th>Covariance Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{T}_I &gt; \mathbf{T}_I^*$</td>
<td>$\mathbf{b}_k = \mathbf{c}_1 \mathbf{T}_I + \mathbf{c}_2$</td>
<td>$\mathbf{R}_{b,k}$ given by algorithm 1</td>
</tr>
<tr>
<td>$\mathbf{T}_I &lt; -\mathbf{T}_I^*$</td>
<td>$\mathbf{b}_k = \mathbf{c}_3 \mathbf{T}_I + \mathbf{c}_4$</td>
<td>$\mathbf{R}_{b,k}$ given by algorithm 1</td>
</tr>
<tr>
<td>$</td>
<td>\mathbf{T}_I</td>
<td>&lt; \mathbf{T}_I^*$</td>
</tr>
</tbody>
</table>

The effect of the BCM on the prediction error can be seen in figure 4. The black bars represent the NRMSE for each sensor when the BCM is turned on. Obviously, the simple model defined above enhances the accuracy of the prediction provided by the on-board model as all NRMSE are closer to unity than when the BCM is disabled (grey bars).

Test-case A: Diagnosis at Test Bench

To assess the improvements brought by the Bias Compensation Module, the following test-case has been developed: it is representative of a maintenance session on a test bench for which sea-level static, standard day conditions are assumed. The evolution of the fuel flow with respect to time is sketched in figure 6. It is an 800-second sequence made of two successive power sweeps between idle and max-continuous regimes, followed by an acceleration between idle and part-power regimes.

For the present application, a subset of six health parameters has been considered: it is made of coefficients affecting the efficiency of the turbo-components (fan, lpc, hpc, hpt and lpt) and a coefficient modifying the nozzle outlet area (see figure 3). The simulated fault consist in a 2-percent loss on the hpt efficiency and is introduced at time $t = 0\ s$.

The evolution of the health parameters identified with the original DEKF (i.e. with the BCM disabled) is plotted in figure 7. The health parameters exhibit an erratic behaviour. Clearly, no valuable information about the health condition of the engine can be derived from the graphs. The health parameters are used by the DEKF as tuners to drive the residuals to zero (on average).

When enabling the BCM, the identification of the health parameters looks as depicted in figure 8. The improvement with respect to the disabled-BCM case is obvious. The health parameters do not wander according to the transient, but converge to their actual value so that at any time, the actual condition of
the engine can be stated. The spikes that can be noted at the beginning of the identification process (first 100 seconds) are actually due to the marginal estimation formulation of the coupled state–health estimation problem (see [10] for more details). The wavelets that can be noted for SE26 and SE41 are due to the small prediction error remaining on \( T^0 \) (see figure 4).

To underline the originality of our approach which models the mean bias level, but also the uncertainty associated to this bias (through the covariance matrix \( R_{b,k} \)), figure 9 depicts the identification of the hpt degradation when using a hybrid BCM setting. The mean bias is computed based on the model presented in table 4, but the covariance matrix of the measurement is set to \( R_{b,k} = R_y \) whatever the TI value. Doing so, no information about the accuracy of the bias \( b_k \) is transmitted to the DEKF. This corresponds to the assumption according which \( b_k \) perfectly catches the model bias.

![Figure 9. DIAGNOSIS OF THE HPT FAULT WITH HYBRID BIAS COMPENSATION MODULE](image)

These results are much better than those presented in figure 7 where the BCM was totally disabled. This hints to the fact that the mean bias has indeed the most disruptive effect on the diagnosis algorithm. Yet, some instabilities are still present in figure 9 which disappear when the covariance matrix \( R_{b,k} \) is used (i.e. in figure 8). This can readily be seen by comparing the smoothness of the curves between figures 9 and 8, or the oscillations of SE49 in figure 9.

From this last result it can be concluded that, even if the modelling of the bias is quite simple and not always very accurate, taking the uncertainty on the bias into account can help in keeping the diagnosis reliable. Hence, the role of the covariance matrix \( R_{b,k} \) in the Kalman filter algorithm is to de-emphasise the influence of the residuals on the health parameters update when a model bias is likely to be expected or when our knowledge about the bias is not very accurate.

**Adapting the BCM to non-SLS Conditions**

In the previous subsection, the positive effect of the BCM on the diagnosis algorithm has been demonstrated. The diagnosis was performed in the same atmospheric conditions as for the extraction of the bias model (i.e. SLS conditions). Practically, it is indeed difficult to collect biases outside of a pass-off test. However, it would be highly valuable to use the predefined BCM for any other ambient conditions.

The extension of the BCM to the whole flight envelope of the engine is nearly immediate by having recourse to the concept of corrected parameters which relies on the theory of similarity. The general expression for correcting parameter \( X \) is given by:

\[
X^{co} = \frac{X}{\theta^a \delta^b}
\]

where \( \theta = T^0/\text{ref}, \delta = p^0/\text{ref}, T_{\text{ref}} = 288.15 \text{ K} \) and \( P_{\text{ref}} = 101325 \text{ Pa} \).

Table 5 reports the exponents \( a \) and \( b \) predicted by the theory of similarity for the different variables involved in the BCM, that is spool speeds, temperatures and pressures on output and spool accelerations and thermal state derivative on input. In regard to the thermal state derivative, the authors derived the reported values by a mixed theoretical–empirical analysis, whereas the values for the other variables can be found in many gas turbine reference books, see for instance [17].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spool Speed</td>
<td>( N )</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>Temperature</td>
<td>( T^0 )</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Pressure</td>
<td>( p^0 )</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Spool Acceleration</td>
<td>( N )</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Thermal State Derivative</td>
<td>( T_m )</td>
<td>0.85</td>
<td>0.8</td>
</tr>
</tbody>
</table>

The following operations allow the adaptation of the original BCM to any ambient conditions:

1. get current state derivatives from the on-board model
2. compute corrected state derivatives according to table 5
3. build corrected TI from corrected state derivatives according to equation 7
4. call the BCM with corrected TI on input, get \( b_k \) and \( R_{b,k} \) on output
5. “de-correct” \( b_k \) to current ambient conditions according to table 5
As can be noted in item 5, the mean bias $b_k$ is “de-corrected”, but not its associated covariance matrix. Hence we implicitly assume that the uncertainty on the mean bias is constant whatever the ambient conditions are. So far no theoretical justification has been found; nonetheless this approach leads to good results in practice as it will be shown in the following. It is also worth mentioning that these adaptations are applied outside of the core of the BCM which is thereby not affected at all.

Test-case B : Validation for non-SLS Conditions

To validate the correction procedure applied to the BCM, a test-case representative of a usual flight sequence from takeoff to cruise conditions was first run. Very accurate results were obtained particularly because the intensity of the transient effects was rather weak, hence minimising the contribution of the BCM.

A second test-case with higher transient levels has then been designed. Cruise flight conditions (altitude = 10801 m, flight Mach number = 0.82) are assumed. The open-loop scheduled fuel flow is plotted in figure 10 and is similar in shape with the one of test-case A. The reader will certainly notice the unreal nature of this test-case which is only intended for validation purpose.

Figure 10. FUEL FLOW PROFILE – CRUISE CONDITIONS

Again, six health parameters are considered in this application. The simulated fault is the same 2-percent degradation of the HPT efficiency, introduced at time $t = 0 \text{ s}$.

Figure 11 sketches the evolution of the identified health parameters when using the corrected BCM. It can be seen that the fault on the HPT is accurately assessed (localisation and magnitude). As for the identification under SLS conditions, one can notice slight oscillations in SE26 and SE41, the cause of which is still the remaining prediction error on $T_0^3$.

Accordingly, it can be stated that the rather simple correction method described above appears to be sufficient to use the original BCM, computed with SLS data, for monitoring the condition of the engine under other ambient conditions. This is a very appealing property of the proposed methodology for bias compensation given that it is much easier to collect biases on the test bench than in flight.

Finally, a fine tuning of the $a$ and $b$ exponents for each parameter involved in the BCM can be considered to take into account phenomena that make the engine behaviour deviate from Mach number similarity. Reference [18] provides the necessary background on that topic.

DISCUSSION

The enhancement of the diagnosis capabilities brought by the BCM in the presence of model-plant mismatch has been pointed out in the previous section. However, some more issues have to be discussed to complete the analysis of the results.

The first question is related to the continuity of the bias model with respect to TI. In this paper, three different TI patterns have been defined and both the mean bias $b$ and the covariance $R_b$ are discontinuous between adjacent patterns. No instabilities of the DEKF have been noticed so far. It is supposed that the DEKF is unaware of those discontinuities because it is not sensible to the derivative of the bias model.

Another open question is linked to the complexity of the bias model. A very simple piece-wise linear bias model has been considered in this application and it has shown to be sufficient for providing a rather accurate diagnosis. However, more complex models such as splines or neural networks could be tested. Another research axis concerning the complexity of the bias model is the definition itself of that model. Throughout the paper, it has been assumed that the bias model only depends on TI. A formulation with two input arguments such as TI and its time derivative, or TI and a state index should be investigated. Data mining techniques could be used to this end.

Finally, some more studies are still to be undertaken concerning robustness issues. Those are threefold: first, the en-
hanced diagnosis algorithm has to be validated with the standard set of eleven health parameters (including coefficients on flow capacities) defined in the frame of the OBIDICOTE project. Secondly, the capability of the algorithm to cope with impulsive noise or with real sensor biases is still under development. Thirdly, the applicability of the methodology presented herein has to be verified for other modelling errors such as sensor/actuator dynamics, fluid dynamics effects, airflow and power off-takes or even biased steady-state engine model.

**CONCLUSION**

The ability to perform a reliable diagnosis in transient operation with an imperfect model of a gas turbine has been investigated. A methodology has been developed to compensate the bias induced by model-plant mismatch by treating it as a pseudo gaussian variable. The improvements on the quality of the diagnosis brought by the new algorithm have been demonstrated on simple, but realistic test-cases.

More specifically, it has been pointed out that taking into account both the mean bias and its related uncertainty improves the identification procedure in terms of stability and accuracy while keeping the structure of the bias model rather simple. The Bias Compensation Module, built from data gathered on a test-bench, has also shown very good generalisation properties in order to carry out health monitoring for any ambient conditions. A simple approach relying on corrected parameters solved this issue.

**REFERENCES**


